Effect of two-temperature electron distribution on the Bohm sheath criterion

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The effect of two-temperature electron distribution on the Bohm sheath criterion is studied in a collisionless sheath layer in contact with a perfectly absorbing wall. A modified Maxwell-Boltzmann model is used for the electron distribution functions, which considers the effect that the electrons above the sheath potential energy do not reflect back into the plasma. The present model shows that the Bohm velocity at the sheath edge is much more independent of the hotter species than what was understood previously with a simple Maxwell-Boltzmann electron model. [S1063-651X(96)01611-X]

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In many plasma devices, the electron velocity distribution may not be a simple Maxwellian. Often, the electrons are found to have two-temperature Maxwellian distributions [1–3]. Understanding the effect of two temperature electron distribution on the Bohm speed has been of significant importance because the Bohm speed enters as a boundary condition in plasma modelings, determining the loss rate to the wall.

Previous analytic studies of Bohm sheath in a two-electron-species plasma used a simple Maxwell-Boltzmann distribution to model each electron species in the sheath layer [4,5]. This model may be justified under the assumption that each electron species is in the state of local thermal equilibrium. The model showed that the Bohm speed in a plasma with two-species electrons is equal to that with one electron species plasma whose temperature is equal to the harmonic average of the two temperature.

However, in a thin sheath layer, where the electron mean free path is longer than the sheath thickness, the local thermal equilibrium assumption may not be justified. In order to have a more accurate understanding of the Bohm speed in a plasma with two-temperature electron distribution, a more realistic model for the electron distribution function needs to be considered. For plasmas with a single electron species, this problem has been studied by Self [6]. He noticed that some of the tail electrons, which are traveling toward an absorbing wall with their energy greater than the sheath potential energy, do not bounce back into the plasma (see Fig. 1). He concluded that inclusion of this phenomenon does not significantly affect the Bohm speed due to the smallness of such electron fraction. Thus, the assumption of local thermal equilibrium was approximately validated in a single-electron-temperature problem, as far as the Bohm speed is concerned.

In a plasma with two-temperature electrons, thermal energy of the more energetic electron species may be greater than the sheath potential energy. In this case, the fraction of the lost electrons which are not bounced back by the sheath potential may be significant within the energetic species, and the assumption of local thermal equilibrium may break down for the energetic species. In the present work, we study the two-temperature electron problem with the consideration of the lost electrons, which have greater energy than the sheath potential energy, and are not bounced back by the sheath potential. The model distribution function of the electrons we choose will be based upon the Maxwell-Boltzmann form, with the elimination of the lost electron part in velocity space.

In order to use a kinetic formulation of Bohm sheath criterion [4,7], we follow Ref. [7] and introduce the following dimensionless quantities:

$$y=\frac{m_i v_x^2}{2kT_e}, \quad \chi=-\frac{eU}{kT_e}, \quad n_{e,i}=\frac{N_{e,i}}{N_0}, \quad \xi=\frac{x}{\lambda_D},$$

where y is the x-directional ion kinetic energy normalized to the electron thermal energy kT_e, χ is the electron potential energy normalized to kT_e, v_x is the particle speed toward the wall, n is the particle density normalized to be unity at the sheath edge, and ξ is the space coordinate x normalized to the electron Debye length. In the present work, we set $U=0$ at the sheath edge. The wall is located at $x=0$ and, thus, the sheath edge is at $x=\infty$.

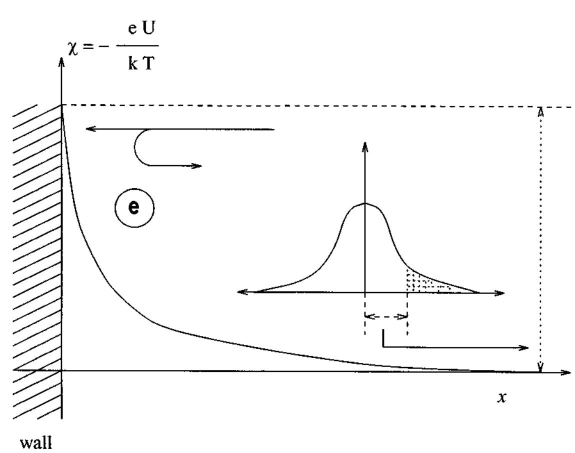

FIG. 1. Truncation of electron distribution function in a nonthermal equilibrium model. Absorbing wall is at $x=0$ and the sheath is formed at $x>0$.

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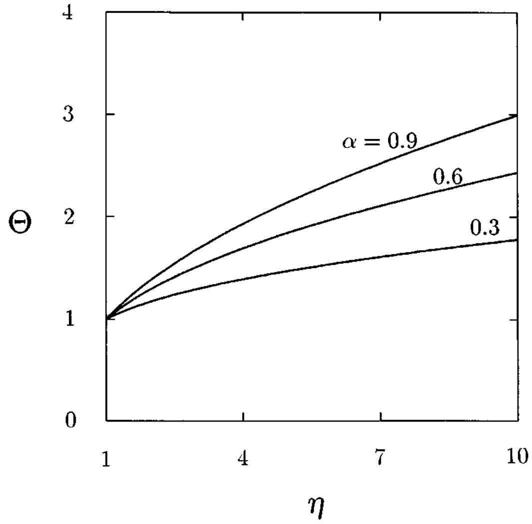

FIG. 2. Effective electron temperature Θ from Boyd and Thompson model as a function of the hot electron temperature normalized to the cold electron temperature.

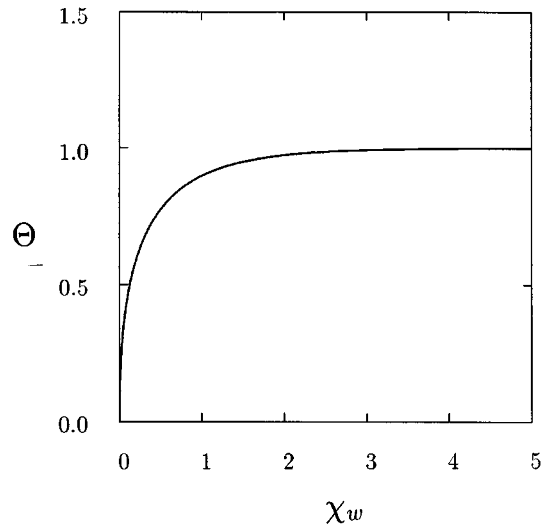

FIG. 3. Effective electron temperature Θ from nonthermal equilibrium model for single-temperature electrons in terms of the normalized sheath potential χ_w.

The Bohm sheath condition can be obtained from the requirement that the electric charge at the sheath edge is positive [7],

$$\rho_0=\left(\frac{dn_i}{d\chi}-\frac{dn_e}{d\chi}\right)\Big|_{\chi=0}\geqslant 0. \tag{1}$$

The quantity $\Theta=-(dn_e/d\chi|_{\chi=0})^{-1}$ can be interpreted as an effective temperature of the total electrons. For the ions, the flow energy is generally dominant over the thermal energy and the concept of effective temperature is not valid.

The above condidtion, Eq. (1), can be easily obtained from Poisson's equation using a similar technique used in Ref. [7],

$$\frac{d^2\chi}{d\xi^2}=n_i(\chi)-n_e(\chi).$$

We multiply this equation by $d\chi/d\xi$ and integrate, with the boundary conditions $\chi,d\chi/d\xi\rightarrow 0$ at the sheath edge (which means that the potential perturbation by the wall fades away at the sheath boundary), to obtain

$$\left(\frac{d\chi}{d\xi}\right)^2=2\int_0^\chi d\chi[n_i(\chi)-n_e(\chi)].$$

We, then, Taylor expand the densities $n_e(\chi)$ and $n_i(\chi)$ near $\chi=0$, and apply the quasineutrality $n_e(0)=n_i(0)$ to obtain

$$\left(\frac{d\chi}{d\xi}\right)^2=2\int_0^\chi d\chi\chi\left[\frac{dn_i}{d\chi}\bigg|_{\chi=0}-\frac{dn_e}{d\chi}\bigg|_{\chi=0}\right]$$
$$=\chi^2\left[\frac{dn_i}{d\chi}\bigg|_{\chi=0}-\frac{dn_e}{d\chi}\bigg|_{\chi=0}\right],$$

near the sheath edge. From this equation we can see that Eq. (1) must be satisfied.

The ion density gradient in χ can be described from a kinetic knowledge we have: If the ion distribution function $f_i(x,v_x)$ is a one dimensional solution to Vlasov equation, then it is a function of energy,

$$f_i(x,v_x)=f_{i0}\left(\sqrt{v_x^2+\frac{2eU}{m_i}}\right) \tag{2}$$

where $f_{i0}=f_i(0,v_x)$. Calculating ion density from f_i, we find

$$n_i=\int f_i(x,v_x)dv=\int f_{i0}\left(\sqrt{v_x^2+\frac{2eU}{m_i}}\right)dv_x$$
$$=\int\left(1-\frac{2eU}{m_iv_0^2}\right)^{-1/2}f_{i0}(v_0)dv_0$$
$$=\int\left(1+\frac{\chi}{y_0}\right)^{-1/2}f_{i0}(v_0)dv_0, \tag{3}$$

where v_0 is v_x at the sheath edge and $y_0=m_iv_0^2/2kT_e$. Then,

$$\frac{dn_i}{d\chi}\Big|_{\chi=0}=-\int\frac{1}{2y_0}f_{i0}(v_0)dv_0=-\frac{kT_e}{m_i}\int\frac{f_{i0}(v_0)}{v_0^2}dv_0. \tag{4}$$

The condition in Eq. (1) can, then, be changed into

$$\langle v_x^{-2}\rangle_{i0}\leqslant\frac{m_i}{\Theta kT_e}, \tag{5}$$

where $\langle\ \rangle_{i0}$ means velocity average over ion distribution function at the sheath edge. When $\Theta=1$, this recovers the usual Bohm sheath criterion in kinetic form.

Information on the effect of two-temperature electron distribution on sheath criterion is contained in the effective temperature. For heuristic purpose, let us first consider the thermal equilibrium case when the electron distribution is simply modeled with two-temperature Maxwell-Boltzmann. If we use the notation $y_e=m_ev_x^2/2kT_e$, we have for the one-dimensional electron distribution function f_e,

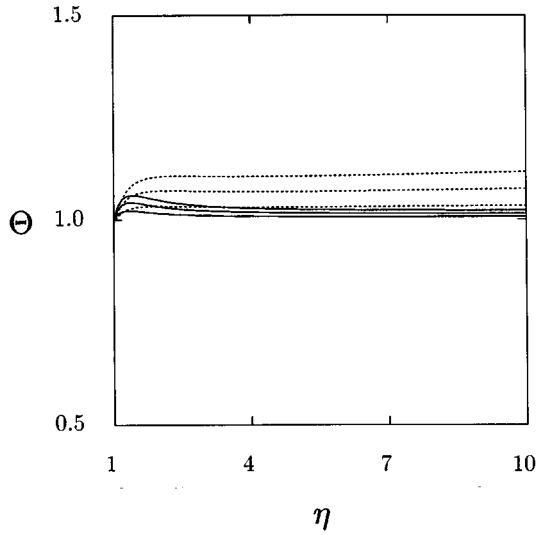

FIG. 4. Effective electron temperature Θ from nonthermal equilibrium model, as a function of the hot electron temperature η normalized to the cold electron temperature. Dotted lines are for $\chi_w=3$ and solid lines are for $\chi_w=5$. Top lines are for $\alpha=0.9$, middle lines for $\alpha=0.6$, and bottom lines for $\alpha=0.3$.

$$f_e(x,v)=C(e^{-(y_e+\chi)}+\alpha e^{-(y_e+\chi)/\eta}), \tag{6}$$

where $\eta(>1)$ is the temperature of the high energy electron species as a ratio to the low energy electron species and C is a normalization constant. Integrating Eq. (6), we get

$$n_e=\int_{-\infty}^{\infty} f_e dv=e^{-\chi}\int_{-\infty}^{\infty} Ce^{-y_e}dv+\alpha e^{-\chi/\eta}\int_{-\infty}^{\infty} Ce^{-y_e/\eta}dv$$

$$=\frac{e^{-\chi}+\alpha\sqrt{\eta}e^{-\chi/\eta}}{1+\alpha\sqrt{\eta}}, \tag{7}$$

which satisfies the normalization condition ($n_e=1$ at $\chi=0$), and yields the effective temperature

$$\Theta=-\left(\frac{dn_e}{d\chi}\bigg|_{\chi=0}\right)^{-1}=\frac{1+\alpha\sqrt{\eta}}{1+\alpha/\sqrt{\eta}}. \tag{8}$$

This, together with Eq. (5), was the previous sheath criterion discussed in the literature [4,5,7] for plasmas with two-electron components under the assumption of local thermal equilibrium. The behavior of Θ as a function of η is shown in Fig. 2 for a few different values of α.

It should be noticed that, in comparing the above kinetic result with those of Refs. [4], [7], and [5], the density of the hot component normalized to the cold component in the present model is $\alpha\sqrt{\eta}$ at the sheath edge. Then, the harmonic average of two temperature with density weighting,

$$\frac{n_c+n_h}{\Theta T_e}=\frac{n_c}{T_e}+\frac{n_h}{T_h},$$

yields Eq. (8), where n_c is the density of cold electron component, $n_h=n_c\alpha\sqrt{\eta}$ is the density of hot electron component and $T_h=\eta T_e$ is the temperature of the hot electron component. Thus, the Bohm speed from a two-Maxwellian model is equal to that with one electron species plasma where temperature is equal to the harmonic average of the two temperatures.

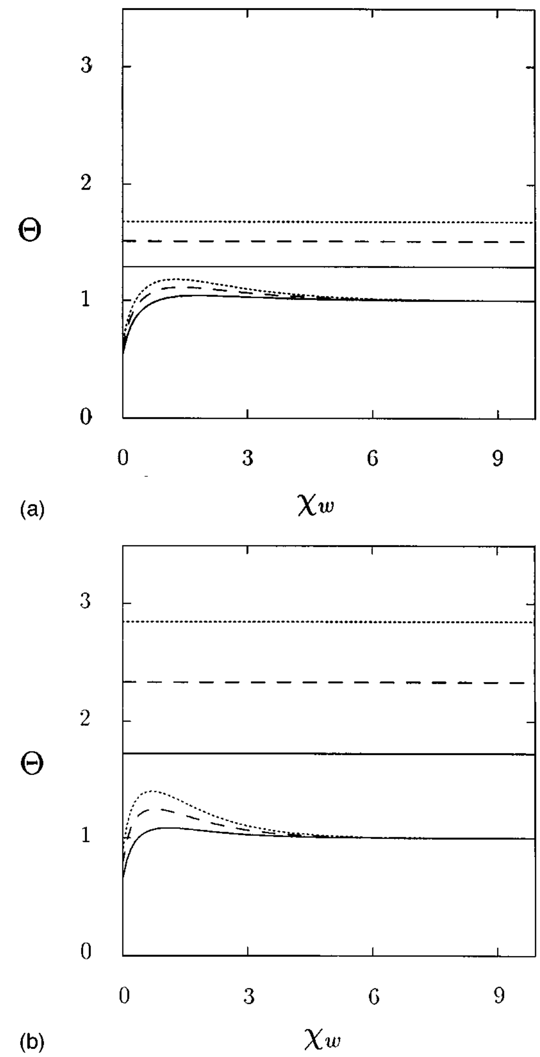

FIG. 5. Effective electron temperature Θ as a function of sheath potential χ_w (a) for $\eta=3$ and (b) for $\eta=9$. Straight lines represent the results of Boyd and Thompson model, and curved lines represent the present results. Dotted lines are for $\alpha=0.9$, dashed lines for $\alpha=0.6$, and solid lines for $\alpha=0.3$.

However, in a collisionless sheath, the electrons flowing to an absorbing wall with their energy above the sheath potential energy will not bounce back by the sheath potential. Thus, among the electrons traveling back into the plasma, the tail part of distribution function corresponding to the electrons above the sheath potential energy should be truncated. Self [6] discussed this truncation effect in the case of a single electron species, concluding that it is small due to the fact that the truncated fraction of the electron density is small at a practical level of sheath potential energy. However, the same cannot be guaranteed for the double-electron species case, in which the thermal energy of the hot species may be above the sheath potential energy. Thus, a large fraction of the hot electron species may be lost to the absorbing wall, not coming back into the plasma. The assumption of local thermal equilibrium may now become a poor choice for the hot electron species.

In order to put the truncation effect into the electron distribution functions, we choose the following model for the electron distribution functions. We let χ_w denote the normalized sheath potential at the wall. If an electron is traveling to the wall with higher energy than χ_w, then it will overcome the sheath potential and get absorbed at the wall (usually, recombine with ions). Thus, the part of the electron distribution function corresponding to the incoming electrons toward the plasma at above the sheath potential energy χ_w needs to be truncated. The one-dimensional electron distribution function may then be modeled as

$$f_e=\begin{cases} 0 \quad \text{if } v_x>0 \text{ and } y_e+\chi>\chi_w \\ C(e^{-(y_e+\chi)}+\alpha e^{-(y_e+\chi)/\eta}) \quad \text{otherwise} \end{cases} \tag{9}$$

where $v_x>0$ corresponds to the direction toward the plasma. Integrating this f_e in the velocity space, it is straightforward to obtain

$$n_e=\frac{e^{\chi_w-\chi}[1+\text{erf}(\sqrt{\chi_w-\chi})]+\alpha\sqrt{\eta}e^{(\chi_w-\chi)/\eta}[1+\text{erf}(\sqrt{(\chi_w-\chi)/\eta})]}{e^{\chi_w}[1+\text{erf}(\sqrt{\chi_w})]+\alpha\sqrt{\eta}e^{\chi_w/\eta}[1+\text{erf}(\sqrt{\chi_w/\eta})]}. \tag{10}$$

From this $n_e(\chi)$, we can calculate a new effective temperature

$$\Theta^{-1}=\frac{e^{\chi_w}[1+\text{erf}(\sqrt{\chi_w})]+\alpha/\sqrt{\eta}e^{\chi_w/\eta}[1+\text{erf}(\sqrt{\chi_w/\eta})]+(1+\alpha)/(\sqrt{\pi\chi_w})}{e^{\chi_w}[1+\text{erf}(\sqrt{\chi_w})]+\alpha\sqrt{\eta}e^{\chi_w/\eta}[1+\text{erf}(\sqrt{\chi_w/\eta})]}. \tag{11}$$

In the present model, Eq. (11) contains all the new information about the effect of two-temperature electrons on the Bohm speed. The case of single electron temperature can be studied by setting $\alpha=0$, and the result is shown in Fig. 3. It can be seen that for a reasonable value of sheath potential $\chi_w\geqslant 2$, the effect of nonthermal equilibrium on Θ (thus, on the Bohm speed) is small. This confirms the result of Ref. [6]. It can also be seen that if there is an external interference (e.g., wall biasing) to reduce the sheath potential below $\chi_w=2$, then the nonthermal equilibrium effect on the Bohm speed can reduce the Bohm speed significantly.

The case of two-temperature electrons ($\alpha>0$) is plotted in Fig. 4 as a function of η for different α values, where η is the hot temperature normalized to the cold temperature. The sheath potential χ_w is set at three (dotted line) and five (solid lines) times the lower electron temperature. By comparing Fig. 4 and Fig. 2, it can be seen that the difference from the thermal equilibrium model (Fig. 2) is significant. The effective temperature Θ in Fig. 4 remains close to the lower electron temperatures, ($\Theta\approx 1$), while that in Fig. 2 is the harmonic average of the two temperatures. What is indicated by the present result is that the existence of high-temperature electron species hardly affects the Bohm speed, much less than what was anticipated by the thermal equilibrium model of Ref. [4].

Figure 5 shows the dependence of effective electron temperature Θ on the sheath potential χ_w. There is a moderate variation of Θ from unity near $\chi_w=1$, and the Bohm speed may have a moderate variation when χ_w is low. However, Θ never recovers the harmonic average value of Ref. [4], shown in Fig. 5 as straight lines, which assumed that both cold and hot electron species are in their own thermal equilibrium states.

We note here that the present result does not indicate that the sheath potential itself is not affected by the nonthermal equilibrium effect. In fact, the escape of hot electron species to the absorbing wall may increase the sheath potential in order to keep the ambipolar flow relation with the ion flow. This problem is presently under investigation by the authors.

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[1] Y. Weng and M. J. Kushner, J. Appl. Phys. **72**, 33 (1992).
[2] D. Vender and R. W. Boswell, IEEE Trans. Plasma Sci. **18**, 725 (1990).
[3] F. Fujita, Jpn. J. Appl. Phys. **29**, 2139 (1990).
[4] R. L. F. Boyd and J. B. Thompson, Proc. R. Soc. A **252**, 102 (1959).
[5] R. W. Boswell, A. J. Lichtenberg, and David Vender, IEEE Trans. Plasma Sci. **20**, 62 (1992).
[6] S. A. Self, Phys. Fluids **6**, 1762 (1963).
[7] K-U. Riemann, J. Phys. D **24**, 493 (1991).